PRACTICAL ASPECTS OF THE MODAL NETWORK REDUCTION TECHNIQUE APPLIED TO THE INTERCONNECTED POWER SYSTEM OF SINGAPORE

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Abstract: A successive modal network reduction technique is outlined and its application to the interconnected power systems of Singapore and Malaysia is demonstrated. The validity of linear, reduced area models over a wide range of disturbances, which have to be studied by utilities on a regular basis is analyzed and discussed. Besides the obvious advantages of drastically reduced simulation times and the option of anonymous data exchange on numerical matrix level with neighboring interconnected systems authorities, the spin-off effect of the presented method is the modal information which is gained by the reduction process itself, allowing for detailed analysis of the modal patterns and oscillatory stability quantities.

Keywords: Power system simulation, large scale systems, system order reduction, computational methods

1. INTRODUCTION

Any type of stability analysis applied for system planning, operation or optimization purposes requires detailed representation of the power plants, loads and transmission networks. Users which perform such studies on a regular basis are often facing the burden of handling and processing data from neighboring networks under other authorities. Excessive computation times and therefore long data processing cycles due to the requirements for a full dynamic and updated representation of the neighboring interconnected systems represent also a considerable drawback.

In order to overcome these problems, the following solutions are typically applied:

- extensive use of special hardware featuring short computation times (Akimoto and co-workers (1990));
- dynamic system reduction based on intuitive simplification methods (Welfonder (1986));
- application of analytical methods like 'Singular Perturbation Techniques' and 'Coherency Based Network Reduction'. See Sauer and co-workers (1987) and Podmore (1978).

The use of fast computer hardware does not simplify in any way the data follow-up and exchange with the neighboring interconnected system authorities. The reported analytical reduction methods are briefly summarized in the followings.

The '**Singular Perturbation Technique**' is based on a time scale separation procedure, which, applied to the describing differential equations of a power system, will represent for fast transients the slow variables as constants, while by the time the changes of the slow transients become observable, it is assumed that the fast transients have already reached their steady state. The weak point of this method is that the transformation of the fast transients into algebraic equations does not guarantee that the dominant poles will be maintained leading by that to stability assessment not on the safe side due to the missing invariance of the modal quantities.

The '**Coherency Based Reduction Technique**' identifies and eliminates power plants which, after a given fault, will oscillate in phase and can therefore be reduced to one single power station. The limitation of this technique is determined of course by the procedure itself, which has to be based essentially on a predefined fault. Consequently, the reduced system can only be valid for the analysis of a certain class of faults and only for a specific fault location.

The above mentioned drawbacks are eliminated in the **successive Modal system order Reduction Technique (MRT)** which has been developed and adapted for the application in power systems investigations; see Weber (1990) and Schmieg (1991).

The present paper is organized as follows. The mixed complete and modal system representation is presented in Section 2. The successive modal-based system order reduction technique (MRT) is explained in Section 3. Practical aspects of the presented reduction technique and the gained modal information are discussed in Section 4. In Section 5 the reduction technique is applied to the Interconnected Power System of Singapore and Malaysia.

2. SYSTEM REPRESENTATION

A commonly used method to fully represent electromechanical power system dynamics for system simulation purposes is based on an iterative procedure which balances the nodal grid against device currents according to Kirchhoff's law for each simulation time step (Schmieg, 1991). This solution method is most flexible and gives unlimited possibilities to include System Devices (SD)described as positive or negative current sources $\underline{i}_{SD}$ based on any kind of mathematical representation, like e.g. synchronous and asynchronous machines, nonlinear loads, reactive compensation devices, HVDC-links as well as multiple fault currents $\underline{i}_f$ in any symmetrical or asymmetrical combination. In this form, a power system is fully represented by a set of algebraic and differential equations:

$$\begin{aligned} \underline{i}_N &= \underline{\underline{Y}}_N \underline{u}_N \\ \underline{i}_N - \Sigma \underline{i}_{SD} - \underline{i}_f &= \underline{0} \end{aligned} \tag{1}$$

and

$$\begin{aligned} \dot{\underline{x}}_{SD_i} &= f\ (\underline{u}_{N_i}, \underline{x}_{SD_i}) \\ \underline{i}_{SD_i} &= g\ (\underline{u}_{N_i}, \underline{x}_{SD_i}) \end{aligned} \tag{2}$$

where $\underline{x}_{SD}$ represent the system states of each dynamic system device. A system area which has to be reduced based on the modal reduction technique, is generally defined by a set of buses which are continuously interconnected and linked to the remaining system via boundary buses. This definition can be applied for areas which are connected at the system border (Fig. 1a), as well as to areas located inside of a power system (Fig. 1b).

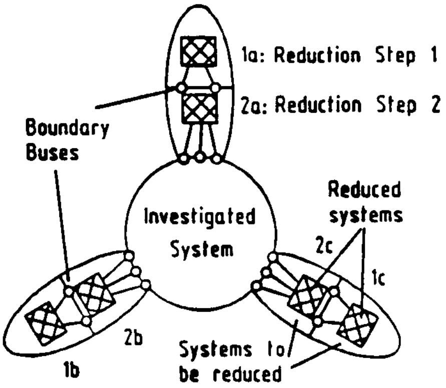

Fig. 1a Definition of area- and boundary quantities for areas connected to the system boarder

The bus current equation on the boundary buses can then be expressed as:

$$\underline{i}_N - \Sigma \underline{i}_{SD} - \underline{i}_f - \underline{i}_{R0} - \Delta \underline{i}_R = \underline{0} \tag{3}$$

where $\underline{i}_{R0}$ denotes the vector of the complex boundary currents resulting from the prefault active and reactive tie-line flow. All buses and system devices inside the defined area have to be replaced by their linear state space equations resulting in:

$$\begin{aligned} \Delta \dot{\underline{x}}_{SD} &= \underline{\underline{A}}\, \Delta \underline{x}_{SD} + \underline{\underline{B}}\, \Delta \underline{u}_R^* \\ \Delta \underline{i}_R &= \underline{\underline{C}}\, \Delta \underline{x}_{SD} + \underline{\underline{D}}\, \Delta \underline{u}_R \end{aligned} \tag{4}$$

where $\Delta \underline{i}_R$ and $\Delta \underline{u}_R$ describe the [2^*n_R] tie-line currents and boundary bus voltages respectively. The extended [2^*n_R+1] bus voltage vector $\Delta \underline{u}_R^*$ is defined as follows:

$$\begin{aligned} \Delta \underline{u}_R^* = [\Delta u_{RR1}, \Delta u_{RI1}; \Delta u_{RR2}, \Delta u_{RI2}; \ldots \\ \ldots\ \Delta u_{RRn}, \Delta u_{RIn};\ \omega_{ref}] \end{aligned} \tag{5}$$

where the reference speed ω_{ref} has been introduced to extend the linear model validity of the reduced system to a certain class of transient stability problems discussed under section 4.

The [$n_R{}^*n_R$] direct input/output-matrix $\underline{\underline{D}}$ is simply defined by the self- and mutual short circuit impedances of the boundary buses. Since the boundary buses themselves will not be affected by the reduction process, the direct input/output-matrix $\underline{\underline{D}}$ will remain unchanged for any reduced system and will therefore represent correctly the subtransient initial short circuit current for any type of fault in the unreduced system as:

$$\Delta \underline{i}_R'' = \underline{\underline{D}}\, \Delta \underline{u}_R \quad ; \quad \Delta \underline{x} = \underline{0} \tag{6}$$

The complex elements of the output-matrix $\underline{\underline{C}}$ determine the direct influence of the appropriate system device state variables with respect to the tie-line currents. In general the matrix $\underline{\underline{C}}$ is a spare matrix, since only a few system devices, especially the synchronous and asynchronous machines, can directly influence the tie-line currents. Based on the known matrix $\underline{\underline{D}}$, the elements c_{ij} of the output-matrix $\underline{\underline{C}}$ can practically be calculated by an iterative procedure as modified jacobean elements according to:

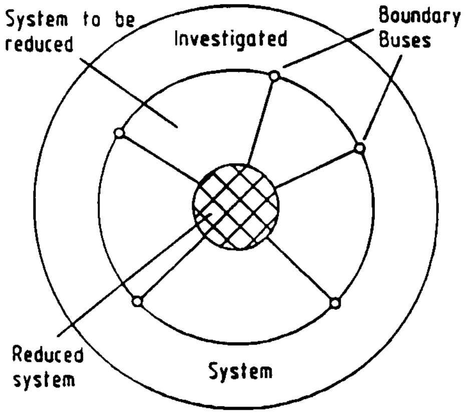

Fig. 1b Definition of area- and boundary quantities for internal areas

$$c_{Rij} + j\, c_{Iij} = \frac{\partial(i_{Ri} + j\, i_{Ii})}{\partial x_j} - D \cdot (\Delta u_{Ri} + j\, \Delta u_{Ii}) \tag{7}$$

The input-matrix $\underline{\underline{B}}$ describes the effect of voltage changes at the boundary buses on the system device states of the considered area. The elements b_{ij} are practically evaluated by an iterative process solving for the Jacobean:

$$b_{Rij} + j\, b_{Iij} = \frac{\partial \dot{x}_i}{\partial u_{Rj} + j\, \partial u_{Ij}} \; ; \quad for \;\; i = 1, n_x$$

$$b_{\omega_i} = \frac{\partial \dot{x}_i}{\partial \omega_{ref}} \; ; \quad for \;\; i = 1, n_x \tag{8}$$

Based on the input-matrix $\underline{\underline{B}}$ the elements a_{ij} of the dynamic matrix $\underline{\underline{A}}$ are calculated according to the structure of eq. (5). The dynamic matrix $\underline{\underline{A}}$ describes the homogenous system of all system devices with respect to a reference generator of the unreduced area. Any transients of the tie-line currents $\Delta \underline{i}_R$ (t) are build up by the superposition of the system modes which are inherent to the dynamic matrix and the direct effects of the boundary bus voltages.

3. MODAL SYSTEM REDUCTION

The basic concept of the MRT, unlike the singular perturbation approach, is not based on the separation and elimination of fast or slow system modes, but on:

- **classification of system states and modes according to their transient dominance with respect to the input-/output behavior at the boundary buses ($\Delta \underline{i} = f(\Delta \underline{u}, \Delta \underline{x})$).**
- **optimal approximation of the transient effects of the reduced states and modes and an exact approximation of the steady state characteristics concerning their input/output behavior at the boundary buses.**

As already stated, the direct matrix $\underline{\underline{D}}$ is not affected by the reduction process, thus leaving the short circuit characteristics of the reduced area unchanged.

The modal system reduction approach is generally subdivided into the following basic steps:

Step 1 Transformation of the system equations (4) to their diagonal form by means of modal transformation techniques.

Step 2 Evaluation of modes according to their dominance with respect to the boundary bus input/output - behavior. The evaluation process itself is based on the calculation of the rated total dominance measure according to:

$$\hat{S}_k = \sum_i \Big(\sum_j \frac{|q_{ikj}|}{|\Delta y_{ij\infty}|} \Big) \quad ; \quad q_{ikj} = \frac{c_{ik}\; b_{kj}}{\lambda_k} \tag{9}$$

Step 3 Approximation of non-dominant modes by optimal, least square based linear combinations of dominant modes. Of course, any unstable mode is dominant and must be included in the essential modes list.

Step 4 Selection of essential state variables with respect to dominant modes according to:

$$Q_{ik} = \sum_j |g_{ikj}| \quad ; \quad g_{ikj} = \frac{v_{ik}\; b_{kj}}{\lambda_k} \tag{10}$$

Alternatively, or in addition, any state variable can be declared as essential if, for any reason, it has to be included in the reduced system.

Step 5 Finally, the reduced system matrices $\underline{\underline{A}}_r$, $\underline{\underline{B}}_r$ and $\underline{\underline{C}}_r$ are calculated by eliminating all non-dominant modes and non-essential states including back-transformation into their original coordinates.

4. PRACTICAL ASPECTS AND MODAL INFORMATION

The efficiency and applicability of the MRT has to be evaluated from most practical aspects of transient power system analysis; before discussing the basic requirements, advantages and limitations, all substantial and characteristic features of the presented reduction technique will be summarized and explained in the followings:

a) An area which has to be reduced by the MRT can be defined without taking care of any dynamic pattern of the area itself or the total interconnected system. This feature allows e.g. for defining areas simply at the boundary buses to other utilities.

b) The MRT is an input- and modal-invariant reduction technique; that is, the reduction procedure is completely independent of any fault type or fault location which will be defined later.

c) The obvious drawback which results from the need of calculating all eigenvalues and right-hand eigenvectors can be solved by applying the MRT successively. As shown in Fig. 1a, the reduced areas themselves may be defined as system devices which can again be included in a further reduction step. By that, sub-areas with approximately 50-100 units represented in detail, or some hundreds of tentatively represented units can be practically considered for each successive reduction step.

d) Areas which have been reduced by MRT represent the steady state and transient characteristics correctly.

For load flow calculations, the boundary bus conditions are:

$$\frac{\partial \Delta \underline{i}_R}{\partial \Delta \underline{u}_R} = (\underline{\underline{D}} - \underline{\underline{C}}\, \underline{\underline{A}}^{-1}\, \underline{\underline{B}}) \tag{11}$$

which define a floating PQ-bus controlling the tie-line flow according to the steady state characteristic of the reduced area. This feature allows tie-line control between reduced and unreduced systems.

For short-circuit calculation or transient contingency analysis, the direct input/output $\underline{\underline{D}}$ matrix can be used since it represents the subtransient self- and mutual impedances at the boundary buses.

e) A remarkable reduction in computation time is a salient feature of the MRT. The system order (number of essential state variables) of reduced areas is typically 5-30. In general, the numerical calculation effort for a reduced area is approximately equivalent to the consideration of one or two fully represented units.

f) A remarkable spin-off effect of the MRT are the dominance- and essential measures inherent to the calculation procedure. In combination with structural pattern analysis, MRT allows for detailed analysis of oscillatory stability problems e.g. optimal localization and parametrisation of stabilizing devices.

g) The accurate dynamic representation of reduced areas at utilities' boundary buses makes possible an anonymous data exchange, which is restricted basically to the four system matrices.

h) The MRT approach can efficiently be implemented in power system analysis programs, since the numerical calculation of the system matrices is based on already defined simulation models (Schmieg, 1991).

Besides the above listed main MRT features, some practical aspects have to be discussed. In the first instance, areas which have been reduced based on MRT are correctly represented with respect to steady-state load flow and initial short circuit current, but the transient behavior is, due to the applied state space description, restricted to small deviations. Consequently, the predestinate application of the MRT featuring practically exact coincidence of original and reduced systems is obviously the oscillatory stability analysis.

On the other side, the applicability of area representation by MRT would be very useful, if transient stability and mid-term stability analysis could also be carried out within acceptable accuracy. The applicability for transient stability analysis is still under investigation, however the following briefly described approach has been already applied successfully.

As shown in Fig.2 a subarea of the system which has to be reduced is defined as a buffer zone between the two systems which will not be taken into consideration for the reduction procedure, but will be fully represented.

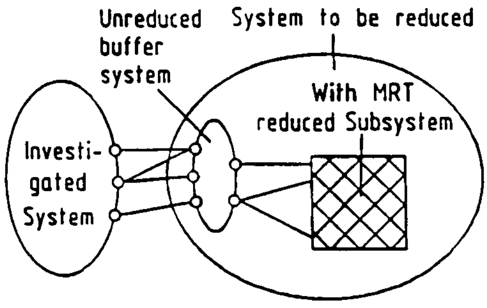

Fig.2 Definition of buffer zone for transient stability analysis

In general, all power stations which are close to the boundary buses should be included in this subarea. Since transient stability characteristics are decisively depending on fault location, this procedure is only needed for faults which are applied on buses electrically close to the reduced area boundary buses.

For mid-term stability analysis the following main non-linearities must be taken into consideration:

- prime mover limitations
- turbine controller limitations and different valve closing and opening times
- frequency-dependent load shedding

It is obvious that these individual non-linearities can generally not be taken into consideration for any reduced system and system reduction method. On the other side, these special information are seldom available and therefore, especially when using predefined simulation models (e.g. IEEE), not considered. Based on these facts, a system reduction for mid-term stability analysis can be performed in following steps:

step 1: Application of the modal reduction technique under the assumption that the prime mover output is constant.

step 2: Splitting of the reduced area current $\Delta \underline{i}_{R}$ into a component $\Delta \underline{i}_{Rel}$ representing the transient current deviation which is based on synchronizing power flow including e.g. voltage control- and power system stabilizer effects and a second current component $\Delta \underline{i}_{Rmech}$ considering the transient effects of prime mover power deviation (e.g. release of spinning reserve).

step 3: Parametrisation and calibration of the equivalent prime mover model based on post mortem analysis.

The above described approach takes into account the practical fact that the electromechanical equivalents derived by the MRT approach are predominantly defined by the well known network and electrical machines quantities, while the prime mover models can be easily and accurately adapted by simple non-linear models based on observed and measured outages.

5. EXAMPLE

The interconnected power system of Singapore and Malaysia, for which the MRT has been applied and tested, is dynamically characterized by a dominant, relatively poor damped interarea-oscillation of a period of approximately 2.3 seconds. The interconnection of both systems is realized via two 230 kV-feeders and associated interbus transformers. The normal operation mode is characterized by zero power-interchange centrally controlled by one of the systems. This relatively poor damped interarea oscillation has made an application of intuitive network reduction techniques impossible, since this dominant dynamic characteristic has to be represented accurately for all relevant system studies.

As indicated in Fig.3, the MRT has been applied to the Malaysian system which is geographically extensiv. However, the Singapore system remains unchanged, ie. unreduced. In this context it is important to mention that the center of the interarea-oscillation is located in the south of the Malaysian system. In consequence, the dynamic pattern of the dominant interarea oscillation shows that all machines of the Singapore system including the southern part of Malaysia will swing in the opposite phase to Middle- and North-Malaysia. Besides this dominant

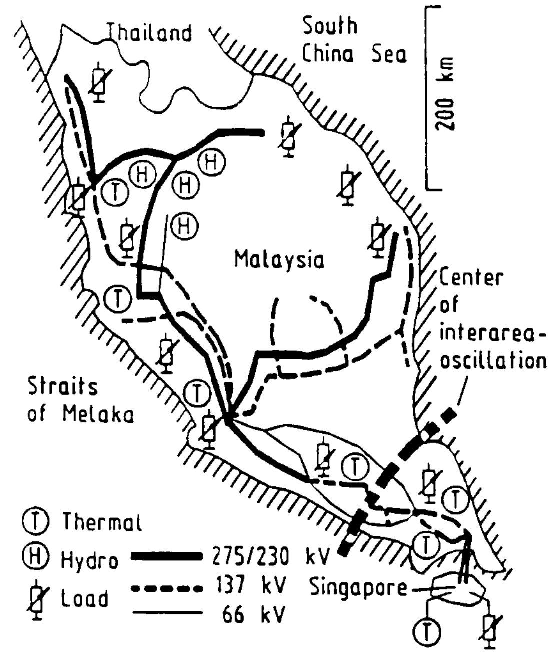

Fig.3 Interconnected power system of Singapore and Malaysia

mode, there exist some other modes with participation of both systems.

The system to be reduced is described by approximately 130 buses, 210 lines, 50 two- and threewinding transformers and 11 detailed represented power stations with more than 200 system state variables and a total generation of approximately 2800 MW which corresponds to a load of nearly 60% compared to the total load of the interconnected system. With the application of MRT, the electrical network of Malaysia was replaced by a set of three reduced 11th-order matrices $\underline{\underline{A}}_r$, $\underline{\underline{B}}_r$ and $\underline{\underline{C}}_r$ and the direct input/output-matrix $\underline{\underline{D}}$ connected directly to the Singapore boundary buses. By that, the total system order of the Malaysian system was reduced by more than 90%.

In order to demonstrate the concurrence of the original and reduced oscillatory system behavior, a torque-impulse defined by:

$$\Delta m_t = \begin{cases} +0.1\ pu & 0.0 \le t < 0.3 \\ -0.1\ pu & 0.3 \le t < 0.6 \\ 0.0\ pu & 0.6 < t \end{cases}$$

has been introduced to the machines located in Singapore. The results of the simulations for both, the reduced and unreduced systems are shown in Fig.4a-d, where especially the active power flow and the boundary bus-voltage and -frequency show practically identical behavior for transient and steady state conditions. Only for the transient reactive power flow some higher differences can be observed.

As a second example, the trip of a fully loaded 250 MW unit located in the power system of Singapore, electrically close to the system boundary buses has been simulated (Fig.5a-d). For this case, a maximum difference of only 2% can be observed between the original and reduced system representation concerning boundary bus-frequency and active power flow, while again a slightly higher mismatch of the boundary bus-voltage and reactive power flow can be found for the initial transients. Of course, for locations inside the Singapore system, the degree of concurrence is significantly higher.

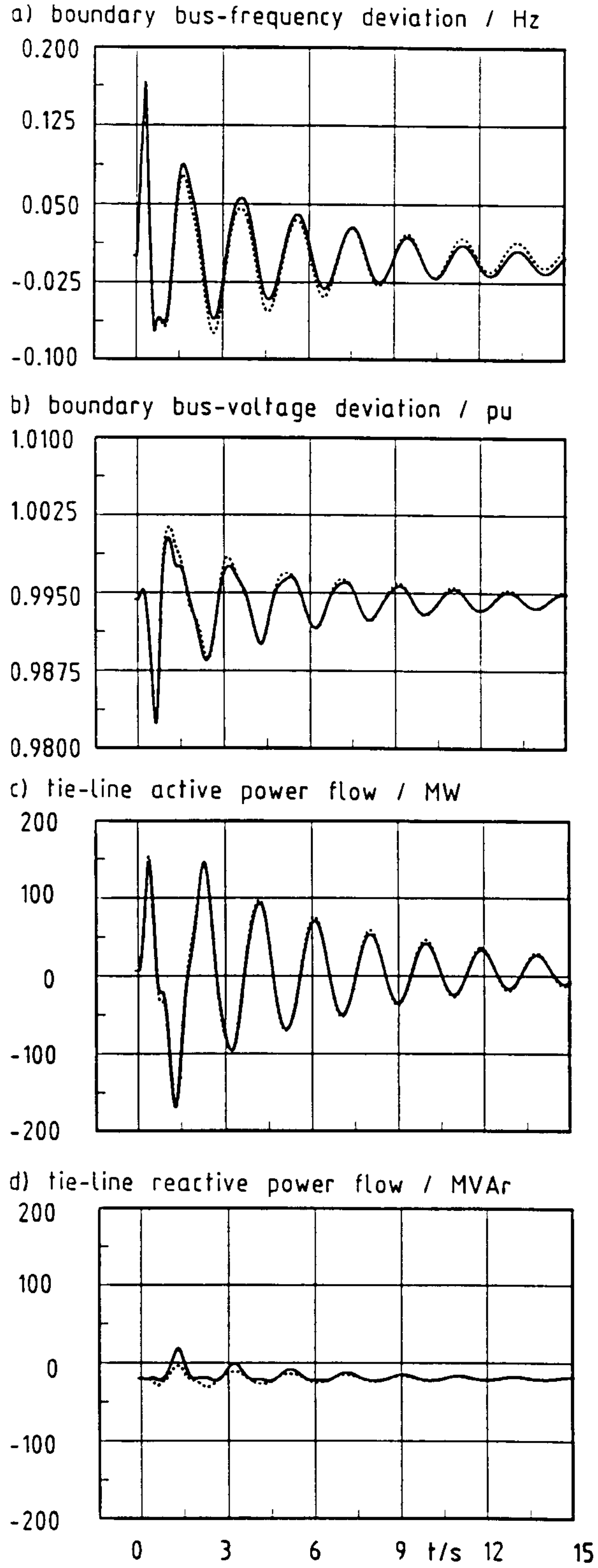

Fig.4 Oscillatory stability behaviour of the original (____) and reduced (.....) power system of Singapore and Malaysia

For both cases, the saving in computation time is more than 60%.

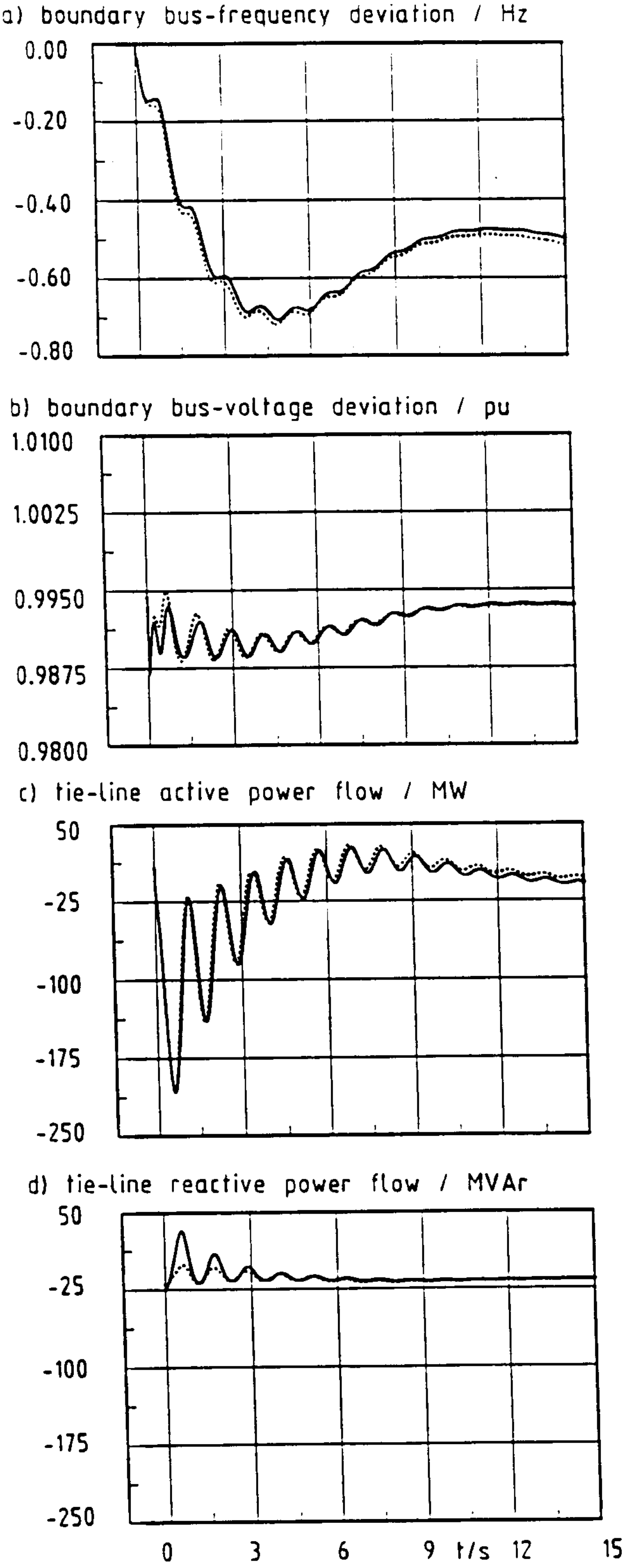

Fig.5 Trip of a 250 MW unit located in the power system of Singapore, original (_____) and reduced (......)

6. CONCLUDING REMARKS

In this paper, the modal reduction technique applied to power systems has been discussed under various practical aspects and applied to the interconnected power system of Singapore and Malaysia. Especially for oscillatory, but also for mid-term stability analysis purposes, the MRT shows significant advantages compared to the other mentioned reduction techniques. Concerning transient stability analysis problems, a systematic computerized procedure to determine the subarea which has to be defined as a buffer zone between the two systems is being studied.

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